Projectile Motion

1

A projectile is launched over level ground and lands some distance away.

a) Is there any point on the trajectory where the velocity v and acceleration a are parallel to each other? If so, where?

no

b) Is there any point on the trajectory where the velocity v and acceleration a are perpendicular to each other? If so, where?

yes

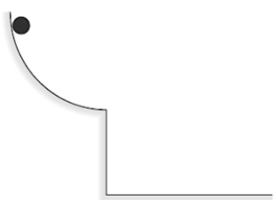
c) Which of the following quantities are *constant* throughout the flight?

x   y   v v_x v_y a_x a_y

3.30 PHYS 21: Chap. 4, Pg 2

2

A ball rolls down a quarter-circle ramp and then off a cliff. Sketch the trajectory of the ball from the instant of its release until it hits the ground.



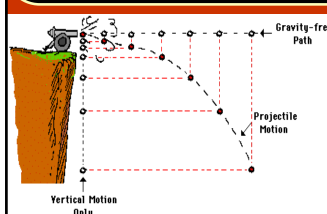
4.9 PHYS 21: Chap. 4, Pg 3

3

ConcepTest 1 Projectile Motion

A ball rolls horizontally off a table, and at the same instant, a second ball is simply dropped from the table? Which ball hits the ground first?

1. the rolling ball hits first
2. the dropped ball hits first
3. both hit at the same time
4. I really have no idea

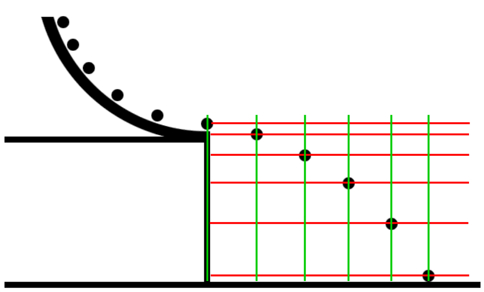


0 of 5

PHYS 21: Chap. 4, Pg 4

4

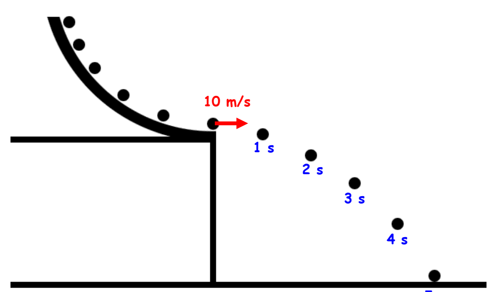
A ball rolls down a quarter-circle ramp and then off a cliff. Sketch the trajectory of the ball from the instant of its release until it hits the ground.



4.9 4, Pg 5

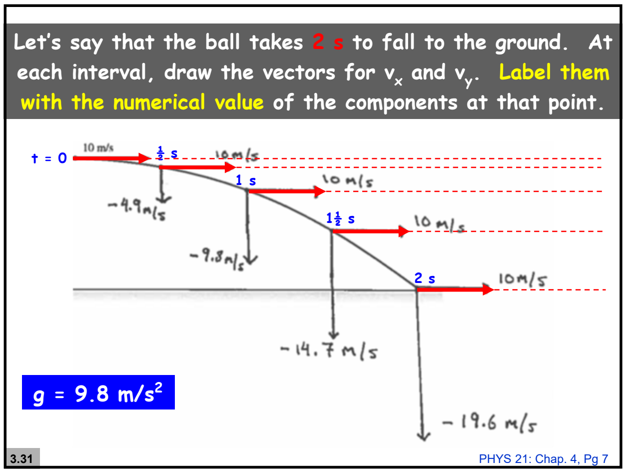
5

Let's say that the ball takes 5 s to fall to the ground. At each interval, draw the vectors for v_x and v_y. Label them with the numerical value of the components at that point.

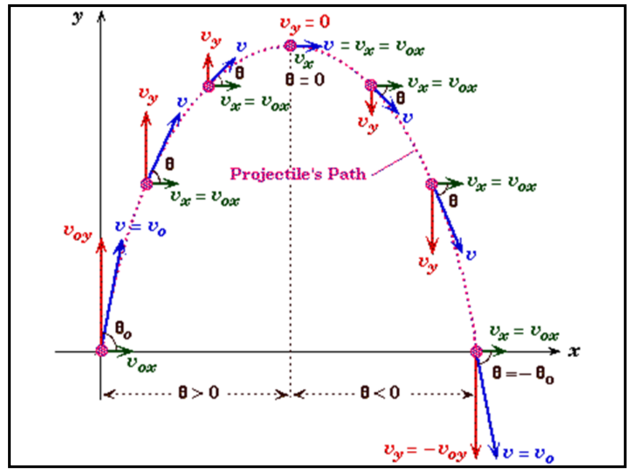


3.31 4, Pg 6

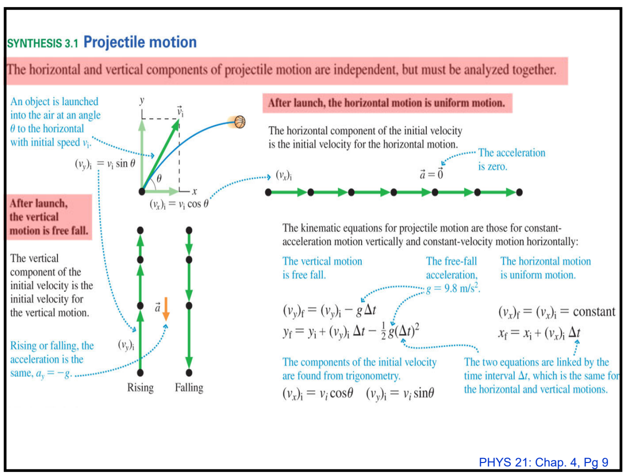
6


Let's say that the ball takes 2 s to fall to the ground. At each interval, draw the vectors for v_x and v_y. Label them with the numerical value of the components at that point.
t = 0
10 m/s
½ s
10 m/s
1 s
10 m/s
1½ s
10 m/s
2 s
10 m/s
−4.9 m/s
−9.8 m/s
−14.7 m/s
−19.6 m/s
g = 9.8 m/s²
3.31
PHYS 21: Chap. 4, Pg 7

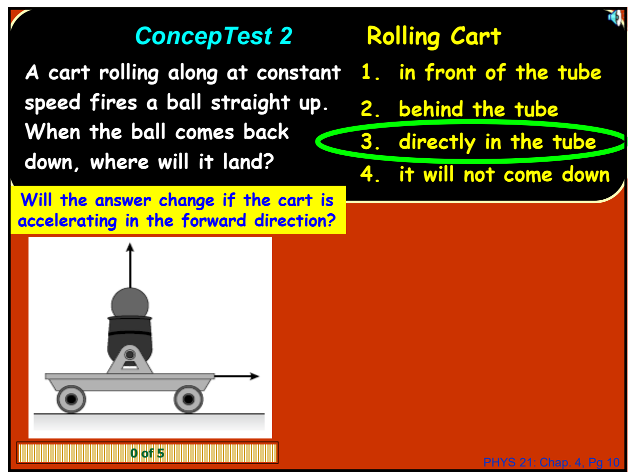
7


y
x
$v_y = 0$
$v = v_x = v_{ox}$
θ = 0
Projectile's Path
v_{oy}
$v = v_o$
$θ_o$
v_{ox}
$v_x = v_{ox}$
θ > 0
θ < 0
θ = −θ_o
$v_y = -v_{oy}$
$v = v_o$

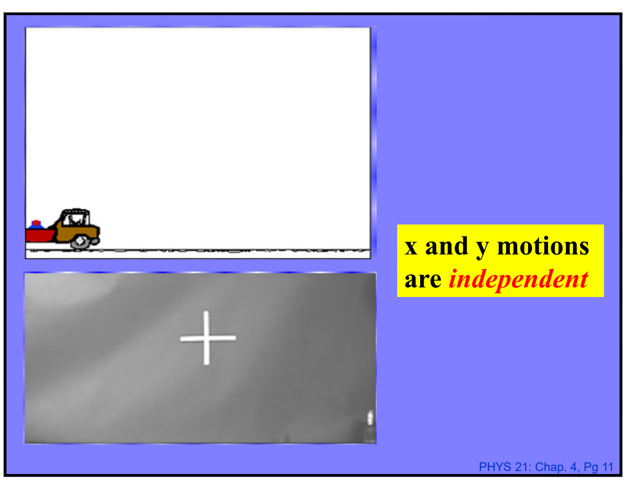
8


SYNTHESIS 3.1 Projectile motion
The horizontal and vertical components of projectile motion are independent, but must be analyzed together.
An object is launched into the air at an angle θ to the horizontal with initial speed v_i.
After launch, the horizontal motion is uniform motion.
The horizontal component of the initial velocity is the initial velocity for the horizontal motion.
The acceleration is zero.
After launch, the vertical motion is free fall.
The vertical component of the initial velocity is the initial velocity for the vertical motion.
Rising or falling, the acceleration is the same, $a_y = -g$.
Rising
Falling
The kinematic equations for projectile motion are those for constant-acceleration motion vertically and constant-velocity motion horizontally:
The vertical motion is free fall.
The free-fall acceleration, g = 9.8 m/s².
The horizontal motion is uniform motion.
$(v_y)_f = (v_y)_i - g\Delta t$
$y_f = y_i + (v_y)_i \Delta t - \frac{1}{2}g(\Delta t)^2$
$(v_x)_f = (v_x)_i = \text{constant}$
$x_f = x_i + (v_x)_i \Delta t$
The components of the initial velocity are found from trigonometry.
$(v_x)_i = v_i\cos\theta \quad (v_y)_i = v_i\sin\theta$
The two equations are linked by the time interval Δt, which is the same for the horizontal and vertical motions.
PHYS 21: Chap. 4, Pg 9

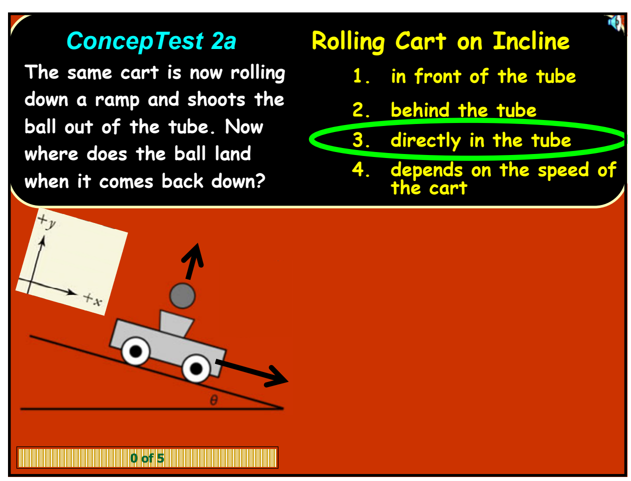
9


ConcepTest 2 Rolling Cart
A cart rolling along at constant speed fires a ball straight up. When the ball comes back down, where will it land?
1. in front of the tube
2. behind the tube
3. directly in the tube
4. it will not come down
Will the answer change if the cart is accelerating in the forward direction?
0 of 5
PHYS 21: Chap. 4, Pg 10

10


x and y motions are independent
PHYS 21: Chap. 4, Pg 11

11


ConcepTest 2a Rolling Cart on Incline
The same cart is now rolling down a ramp and shoots the ball out of the tube. Now where does the ball land when it comes back down?
1. in front of the tube
2. behind the tube
3. directly in the tube
4. depends on the speed of the cart
+y
+x
θ
0 of 5

12

x and y motions are *independent*

Kinematic Equations for Constant Acceleration in 2 Dimensions

x Component (horizontal)	*y* Component (vertical)
$v_x = v_{x0} + a_x t$	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$	$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

Projectile Motion: $a_x = 0$ and v_x = const.

(y positive upward; $a_x = 0$, $a_y = -g = -9.80\ \mathrm{m/s^2}$)

Horizontal Motion ($a_x = 0$, v_x = constant)	Vertical Motion† ($a_y = -g$ = constant)
$v_x = v_{x0}$	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

† If y is taken positive downward, the minus (−) signs in front of g become plus (+) signs.

13

PROBLEM-SOLVING APPROACH 3.1 **Projectile motion problems**

1 **STRATEGIZE** We will solve projectile motion problems by considering the horizontal and vertical motions as separate but related problems.

2 **PREPARE** There are a number of steps that you should go through in setting up the solution to a projectile motion problem:

- Make simplifying assumptions. Whether the projectile is a car or a basketball, the motion will be the same.
- Draw a visual overview including a pictorial representation showing the beginning and ending points of the motion.
- Establish a coordinate system with the x-axis horizontal and the y-axis vertical. In this case, you know that the horizontal acceleration will be zero and the vertical acceleration will be free fall: $a_x = 0$ and $a_y = -g$.
- Draw a vector representing the initial velocity, and find its x- and y-components in terms of the initial speed and the launch angle.
- Define symbols and write down a list of known values. Identify what the problem is trying to find.

3 **SOLVE** There are two sets of kinematic equations for projectile motion, one for the horizontal component and one for the vertical:

Horizontal	Vertical
$x_f = x_i + (v_x)_i\,\Delta t$	$y_f = y_i + (v_y)_i\,\Delta t - \frac{1}{2}g(\Delta t)^2$
$(v_x)_f = (v_x)_i$ = constant	$(v_y)_f = (v_y)_i - g\,\Delta t$

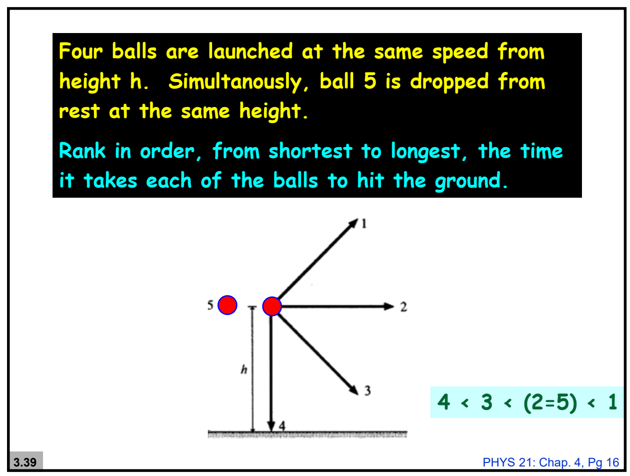
Δt is the same for the horizontal and vertical components of the motion. Find Δt by solving for the vertical or the horizontal component of the motion; then use that value to complete the solution for the other component.

4 **ASSESS** Check that your result has the correct units, is reasonable, and answers the question.

/S 21: Chap. 4, Pg 14

14

15



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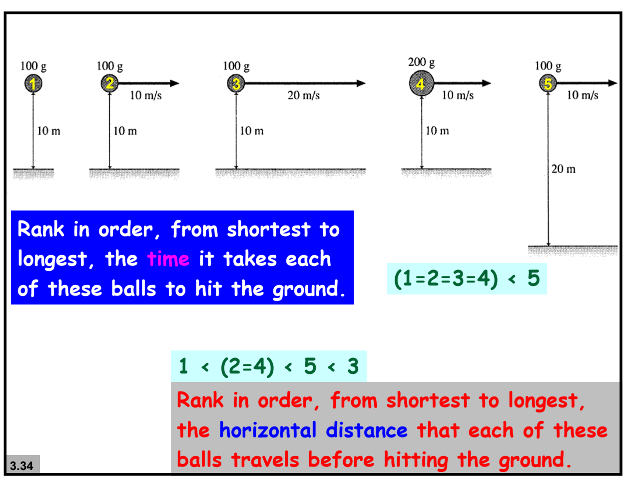

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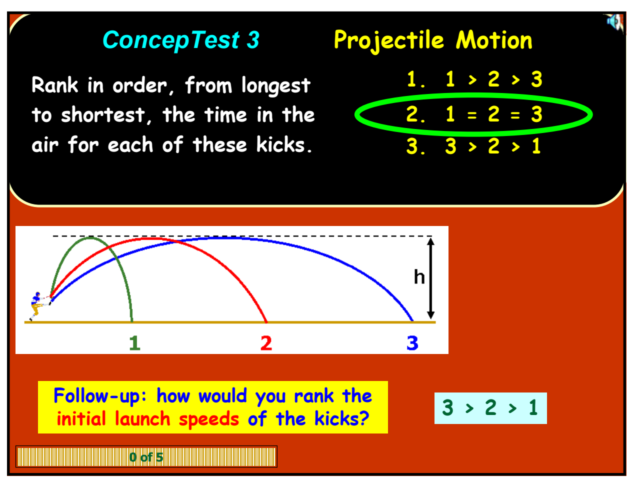
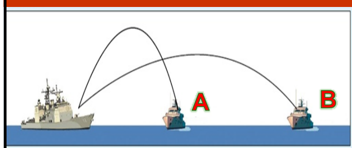


18

ConcepTest 4 **Projectile Motion**

A battleship simultaneously fires two shells at enemy ships, and the trajectories are shown. **Which ship gets hit first?**

1. ship A
2. ship B
3. both at the same time



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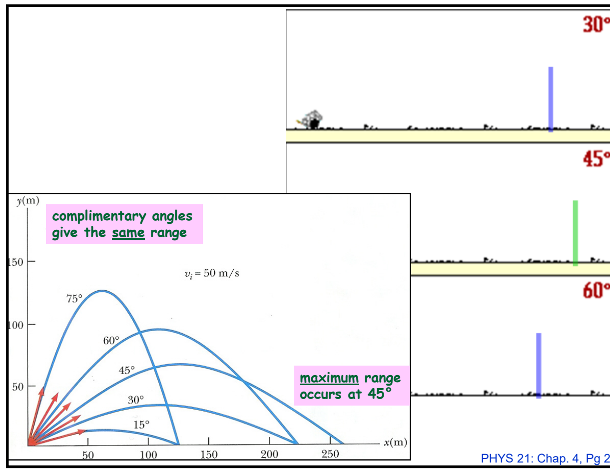


PHYS 21: Chap. 4, Pg 20

20

On the Apollo 14 mission, astronaut Alan Shepard hit a golf ball on the surface of the Moon. The acceleration due to gravity on the Moon is 1/6 that of Earth. Suppose that he hit the golf ball with a speed of **25 m/s** at an angle of **30°** above the horizontal.

(a) How long was the ball in flight? t = 15.3 s

(b) How far did the ball travel? x = 331 m

(c) How far would the ball travel if it had been hit on Earth?

x = 55.3 m

3+2:5 3.34

21

On the Apollo 14 mission, astronaut Alan Shepard hit a golf ball on the surface of the Moon. The acceleration due to gravity on the Moon is 1/6 that of Earth. Suppose that he hit the golf ball with a speed of **25 m/s** at an angle of **30°** above the horizontal.

(a) How long was the ball in flight? t = 15.3 s

(b) How far did the ball travel? x = 331 m

x = 55.3 m

(c) How far would the ball travel if it had been hit on Earth?

$v_x = v(cos\theta) = 21.7\ m/s$ $\quad v_y = v(sin\theta) = 12.5\ m/s$

$v_{fy} = v_{iy} - \left(\frac{g}{6}\right)t_{top}$ → $t_{top} = \frac{-v_{iy}}{\left(-g/6\right)} = 7.65\ s$

$t_{flight} = 2t_{top} = 15.3\ s$ → $\Delta x = v_{ix}t_{flight} = 332\ m$

On Earth, the flight time is 6× shorter. $\quad t_{flight} = 2.55\ s \quad \Delta x = v_{ix}t_{flight} = 55.3\ m$

Why?

3+2:5 3.34

PHYS 21: Chap. 2, Pg 22

22

The longest recorded pass in an NFL game traveled **83 yds** in the air from the quarterback to the receiver. If the pass was thrown at a launch angle of **45°**, what was the speed at which the ball left the quarterback's hand?

x = 27.3 m/s

3+2:5 3.64

PHYS 21: Chap. 2, Pg 23

23

The longest recorded pass in an NFL game traveled **83 yds** in the air from the quarterback to the receiver. If the pass was thrown at a launch angle of **45°**, what was the speed at which the ball left the quarterback's hand? x = 27.3 m/s

$v_{ix} = v(cos\theta) \quad v_{iy} = v(sin\theta)$

$v_{fy} = -v_{iy} = v_{iy} - gt$ → $t = \frac{v_{fy} - v_{iy}}{-g} = \frac{-2v(\sin\theta)}{-g}$

$\Delta x = v_{ix}t = v(\cos\theta)t = \frac{2v^2(\cos\theta)(\sin\theta)}{g} = 83\ yds = 76\ m$

$v^2 = \frac{g(76\ m)}{2(\cos 45)(\sin 45)}$ → $v = \sqrt{\frac{g(76\ m)}{2\,(\cos 45)(\sin 45)}} = 27.3\ m$

3+2:5 3.64

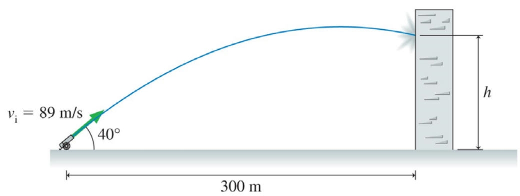
PHYS 21: Chap. 2, Pg 24

24

A cannon is aimed at a **40°** angle and fires at a wall **300 m** away on level ground. If the initial speed of the cannonball is **89 m/s**, at what height *h* does the ball hit the wall?

t = 4.4 s

h = 156.8 m



3+2:3

Q3.23

25

A cannon is aimed at a **40°** angle and fires at a wall **300 m** away on level ground. If the initial speed of the cannonball is **89 m/s**, at what height *h* does the ball hit the wall?

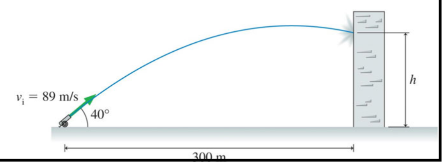
t = 4.4 s

h = 156.8 m

$v_{ix} = v(cos\theta) = 68.2\,\frac{m}{s}$ $\quad v_{ix} = v(sin\theta) = 57.2\,\frac{m}{s}$

$\Delta x = v_{ix}t = 300\,m$ → $t = \frac{300m}{v(\cos\theta)} = \frac{300m}{68.2\,\frac{m}{s}} = 4.4\,s$

$y_f = v_{iy}t - \frac{1}{2}gt^2 = (57.2\,\frac{m}{s})t - \frac{1}{2}gt^2 = 156.8\,m$



3+2:3

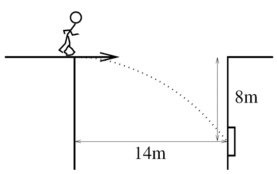
Q3.23

26

In a James Bond movie, our daring secret agent has to deliver a package in a subtle manner. From the roof of one building, he must kick the package *horizontally* off the roof at just the right speed to have the package sail through the window of a neighboring building. The window is located **8 m** below the roof and is **14 m** away horizontally.

What must be the initial speed of the package? v_x = 10.9 m/s

What is the final <u>velocity</u> of the package? v = 16.6 m/s, θ = -48.9°



$v_{fs} = v_{is} + a_s\Delta t$

$s_f = s_i + v_{is}\Delta t + \frac{1}{2}a_s(\Delta t)^2$

$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$

3+2:3

27

In a James Bond movie, our daring secret agent has to deliver a package in a subtle manner. From the roof of one building, he must kick the package *horizontally* off the roof at just the right speed to have the package sail through the window of a neighboring building. The window is located **8 m** below the roof and is **14 m** away horizontally.

What must be the initial speed of the package? v_x = 10.9 m/s

What is the final <u>velocity</u> of the package? v = 16.6 m/s θ = -48.9°

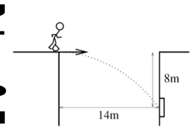
SOLUTION

$y = \frac{1}{2}gt^2$ → $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(8m)}{g}} = 1.28s$

$x = v_x t$ → $v_x = \frac{x}{t} = \frac{14m}{1.28s} = 10.9m/s$

$v_y = v_{0y} + a_y t = 0 - gt = -12.5m/s$

$v = \sqrt{v_x^2 + v_y^2} = 16.6m/s$ $\quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = -48.9°$



28

King Arthur's knights fire a cannon from the top of the castle wall. The cannonball is fired at a speed of **50 m/s** and an angle of **30°**. A cannonball that was accidentally dropped hits the moat below in **1.5 s**.

a) How far from the castle wall does the cannonball hit the ground? x = 238 m

b) What is the cannonball's maximum height above the ground? y_{max} = 31.9+11 m

$v_{fs} = v_{is} + a_s\Delta t$

$s_f = s_i + v_{is}\Delta t + \frac{1}{2}a_s(\Delta t)^2$

$v_{fs}^2 = v_{is}^2 + 2a_s\Delta s$

3+2:3

4.51

29

King Arthur's knights fire a cannon from the top of the castle wall. The cannonball is fired at a speed of **50 m/s** and an angle of **30°**. A cannonball that was accidentally dropped hits the moat below in **1.5 s**.

a) How far from the castle wall does the cannonball hit the ground?

x = 238 m

b) What is the cannonball's maximum height above the ground?

SOLUTION

$x_{0x} = v_0\cos\theta = 43.3m/s$

$x_{0y} = v_0\sin\theta = 25.0m/s$

$y = \frac{1}{2}gt^2 = \frac{1}{2}g(1.5s)^2 = 11.0m$

$y_f = y_i + v_{0y}t + \frac{1}{2}a_y t^2$

$0 = 11 + 25t - \frac{1}{2}gt^2$ → $t = 5.5s$

$x_f = v_{0x}t = (43.3m/s)(5.5s) = 238m$

30

King Arthur's knights fire a cannon from the top of the castle wall. The cannonball is fired at a speed of **50 m/s** and an angle of **30°**. A cannonball that was accidentally dropped hits the moat below in **1.5 s**.

a) How far from the castle wall does the cannonball hit the ground? $x = 238$ m

b) What is the cannonball's maximum height above the ground? $y_{max} = 31.9+11$ m

SOLUTION

$$x_{0x} = v_0 \cos\theta = 43.3 m/s$$
$$x_{0y} = v_0 \sin\theta = 25.0 m/s$$
$$y = \frac{1}{2}gt^2 = \frac{1}{2}g(1.5s)^2 = 11.0m$$
$$y_f = y_i + v_{0y}t + \frac{1}{2}a_y t^2$$
$$0 = 11 + 25t - \frac{1}{2}gt^2 \rightarrow t = 5.5s$$
$$x_f = v_{0x}t = (43.3m/s)(5.5s) = 238m$$

$$v_f^2 = v_i^2 - 2gy$$
$$0 = v_{0y}^2 - 2gy$$
$$y = \frac{v_{0y}^2}{2g} = 31.9m$$
$$y_{max} = 31.9m + 11m = 42.9m$$

31

You are playing left field for the baseball team. Your team is up by one run in the bottom of the 9th inning when a ground ball slips through the infield and comes straight toward you.

As you pick up the ball **65 m** from home plate, you see a runner rounding third base and heading for home with the tying run. You throw the ball at an angle of **30°** above the horizontal with just the right speed so that the ball is caught by the catcher at home plate at the same height as you threw it. As you release the ball, the runner is **20 m** from home plate and running full speed at **8 m/s**.

Will the ball arrive in time for your team's catcher to make the tag and win the game?

$t_{ball} = 2.77$ s

$t_{run} = 2.50$ s

3+2:5

4.48

32

You pick up the ball **65 m** from home plate, you see a runner rounding third base and heading for home with the tying run. You throw the ball at an angle of **30°** above the horizontal with just the right speed so that the ball is caught by the catcher at home plate at the same height as you threw it. As you release the ball, the runner is **20 m** from home plate and running full speed at **8 m/s**. **Will the ball arrive in time for your team's catcher to make the tag and win the game?**

$t_{ball} = 2.77$ s

$t_{run} = 2.50$ s

SOLUTION

$$x = 65m = v_x t = (v_0 \cos\theta)t \rightarrow v_0 = \frac{x}{t\cos\theta}$$
$$y_f = y_i + v_{0y}t + \frac{1}{2}a_y t^2$$
$$0 = 0 + (v_0 \sin\theta)t - \frac{1}{2}gt^2 \rightarrow v_0 = \frac{gt}{2\sin\theta}$$
$$\frac{x}{t\cos\theta} = \frac{gt}{2\sin\theta} \rightarrow t_{ball} = \sqrt{\frac{2x\sin\theta}{g\cos\theta}} = 2.77s$$
$$t_{runner} = \frac{20m}{8m/s} = 2.50s$$

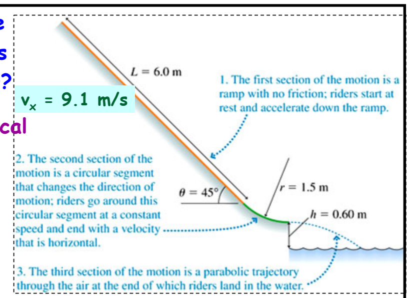
33

1) At the end of the first section, what is the speed of a rider?

$v_x = 9.1$ m/s

2) What is the vertical component of the velocity of a rider as she hits the water at the end?

$v_y = 3.4$ m/s



3) Suppose the accel. in the 2nd section is too large to be comfortable for riders. What can be done to decrease it?

a) reduce the radius of the circular segment

b) increase the radius of the circular segment

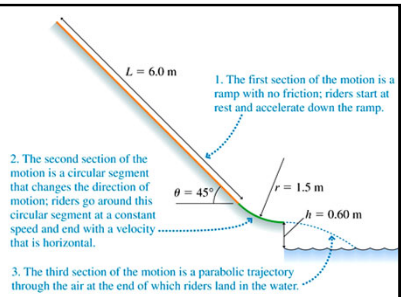
c) increase the ramp angle

d) increase the ramp length

3.78-82

34

4) During which section is the magnitude of the acceleration experienced by a rider the greatest?



5) Suppose the designers of the water slide want to adjust the height h above the water so that riders land twice as far away from the bottom of the slide. What would be the new height h' above the water?

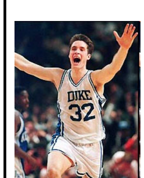
$h' = 4h = 2.4$ m

3.78-82

35

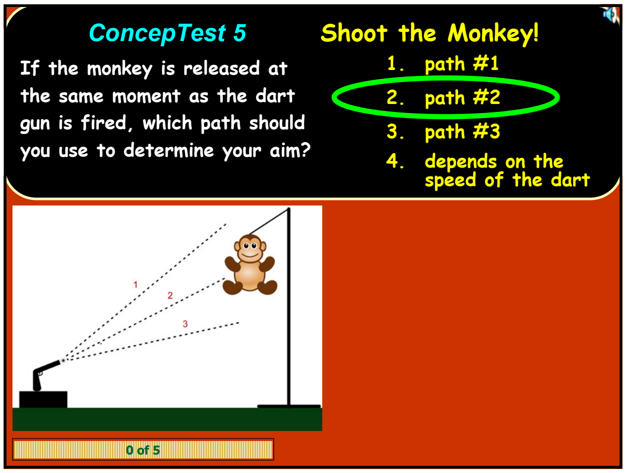
Video Analysis

Projectile Motion



Pasco123 (1D)
Pasco108 (2D)

36



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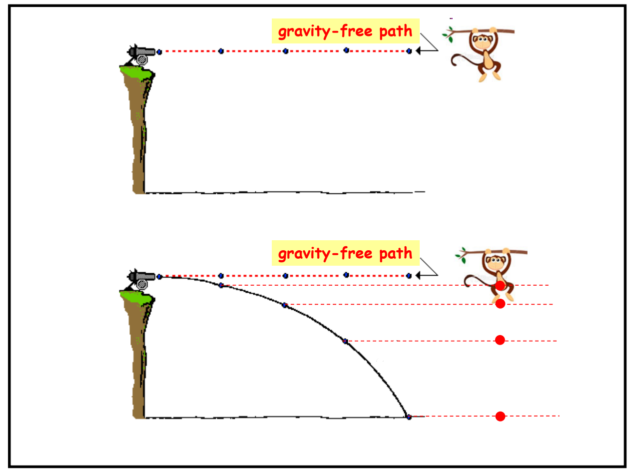

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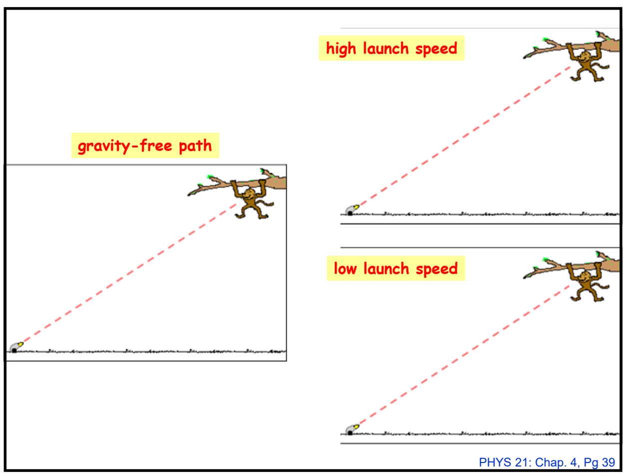

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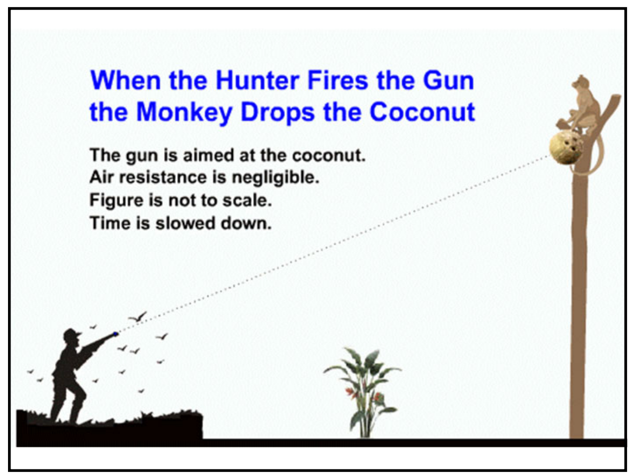

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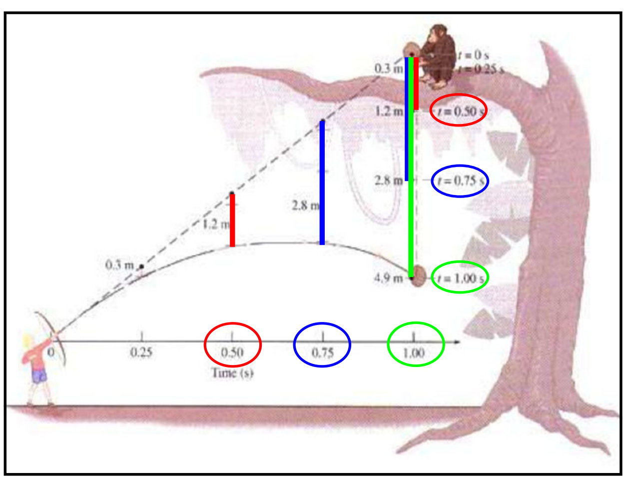

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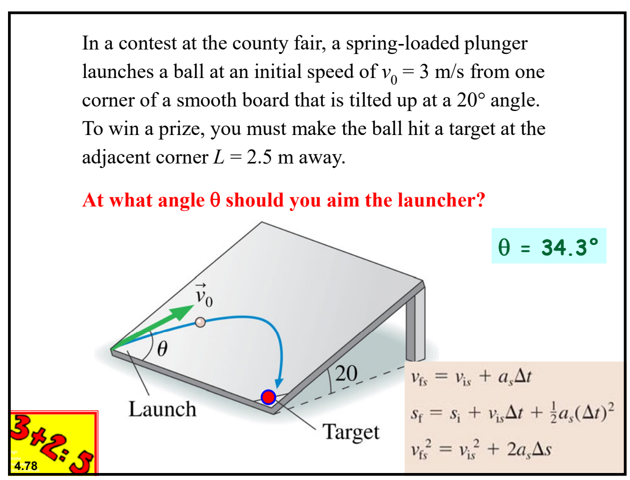

42

In a contest at the county fair, a spring-loaded plunger launches a ball at an initial speed of $v_0 = 3$ m/s from one corner of a smooth board that is tilted up at a 20° angle. To win a prize, you must make the ball hit a target at the adjacent corner $L = 2.5$ m away.

$\theta = 34.3°$

At what angle θ should you aim the launcher?

SOLUTION

$$v_x = v_0 \cos\theta$$
$$v_y = v_0 \sin\theta$$
$$g' = g(\sin 20°) = 3.35 m/s^2$$

$$L = 2.5m = v_x t \rightarrow t = \frac{L}{v_x} = \frac{L}{v_0 \cos\theta}$$

$$y_f = 0 = 0 + v_y t - \frac{1}{2} g' t^2$$

$$0 = (v_0 \sin\theta)\left(\frac{L}{v_0 \cos\theta}\right) - \frac{g'}{2}\left(\frac{L}{v_0 \cos\theta}\right)^2$$

trig identity

$$2 \sin\theta \cos\theta = \frac{g' L}{v_0^2} = 0.93$$

$$\sin 2\theta = 0.93 \rightarrow 2\theta = 68.6° \rightarrow \theta = 34.3°$$

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43

Relative Motion

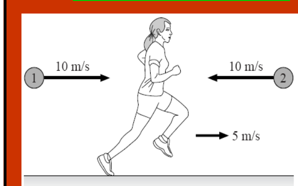
44

ConcepTest 3 Relative Motion

Joan runs to the right at 5 m/s. Balls 1 and 2 are thrown to her at 10 m/s by friends on the ground. According to Joan, which ball is moving faster?

1. both the same
2. ball #1
3. ball #2

To Joan, what are the speeds of the balls?



$v_1' = 5$ m/s

$v_2' = 15$ m/s

0 of 5

PHYS 21: Chap. 4, Pg 45

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ConcepTest 4 Relative Motion

Joan runs to the right at 5 m/s. Balls 1 and 2 are thrown to her by friends on the ground. According to Joan, both balls are approaching her at 10 m/s. Which ball was thrown faster?

1. both the same
2. ball #1
3. ball #2

What are the ground speeds of the balls?

$v_1 = 15$ m/s

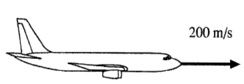
$v_2 = 5$ m/s

0 of 5

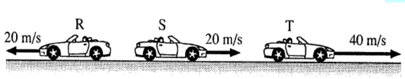
PHYS 21: Chap. 4, Pg 46

46

Ryan, Samantha and Tom are driving in their cars. At the same time, a plane moving at 200 m/s is flying overhead. Rank in order, from largest to smallest, the jet's speed according to each person.



R > S > T



What are these relative speeds?

(R=220) > (S=180) > (T=160)

3.43

PHYS 21: Chap. 4, Pg 47

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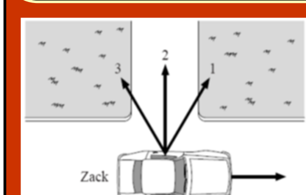
ConcepTest 5 Relative Motion

Zack is driving past his house and wants to toss his physics book into the driveway. How should he direct his throw to accomplish this?

1. path 1
2. path 2
3. path 3



0 of 5

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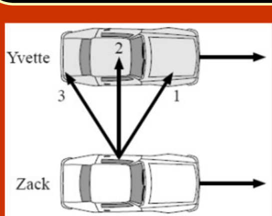
ConcepTest 6 Relative Motion

Zack and Yvette are driving side by side with their windows rolled down. Zack wants to toss his physics book into the front seat of Yvette's car. How should he direct his throw?

1. path 1
2. path 2
3. path 3



0 of 5

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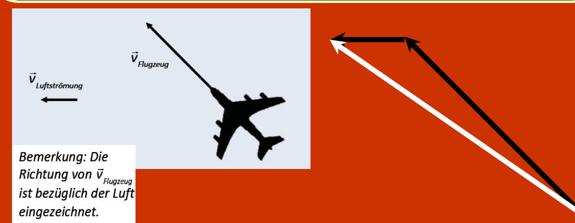
ConcepTest 8 Relative Motion

A jet has an airspeed of 980 km/hr and it flies in a wind of 120 km/hr in a different direction. What is the ground speed of the plane?

1. v = 120
2. 120 < v < 980
3. v = 980
4. 980 < v < 1100
5. v = 1100



0 of 5

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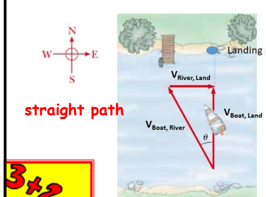
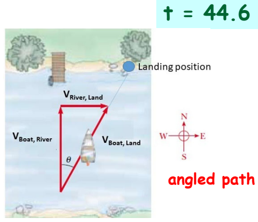
A kayaker wants to paddle north across a 100 m wide river.
The current in the river is flowing to the east at 2 m/s.
The kayaker can paddle in still water at a speed of 3 m/s.

In which direction should the kayaker paddle in order to travel straight across the river? θ = 41.8°

How long will it take the kayaker to cross the river? t = 44.6 s





3+2: 3

4.53

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SOLUTION

A kayaker wants to paddle north across a 100 m wide river.
The current in the river is flowing to the east at 2 m/s.
The kayaker can paddle in still water at a speed of 3 m/s.

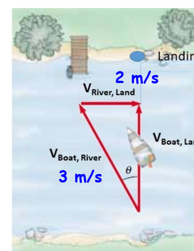
In which direction should the kayaker paddle in order to travel straight across the river?

How long will it take the kayaker to cross the river?

$$sin\theta = \frac{2}{3} \Rightarrow \theta = 41.8°$$

$$v_{boat,land} = 3\ cos\theta = 2.24\ \frac{m}{s}$$

$$t = \frac{\Delta x}{v} = \frac{100\ m}{2.24\ \frac{m}{s}} = 44.6\ s$$



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