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Chapter 4 - Kinematics in Two Dimensions

- Motion in 2D
- Projectile motion
- Relative motion
- Uniform circular motion
 - ✓ angular velocity and angular position
 - ✓ centripetal acceleration
- Non-uniform circular motion
 - ✓ angular vs. tangential acceleration

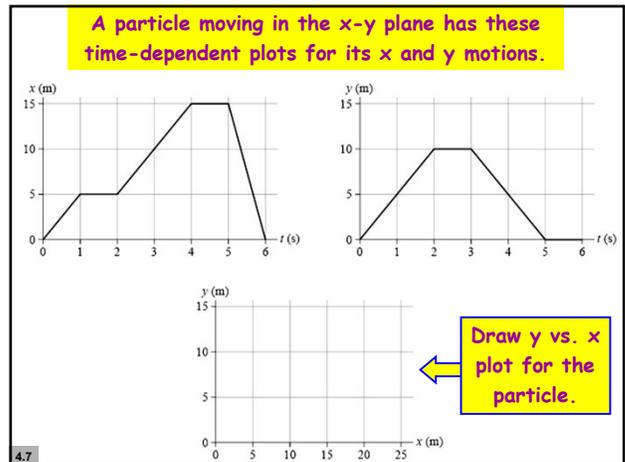
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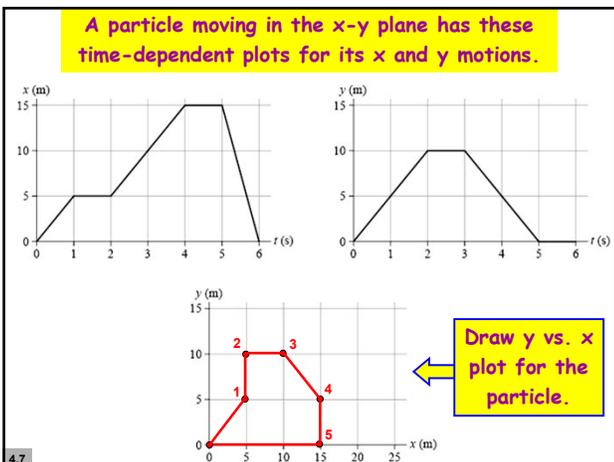
2D Kinematics

(including projectile motion)

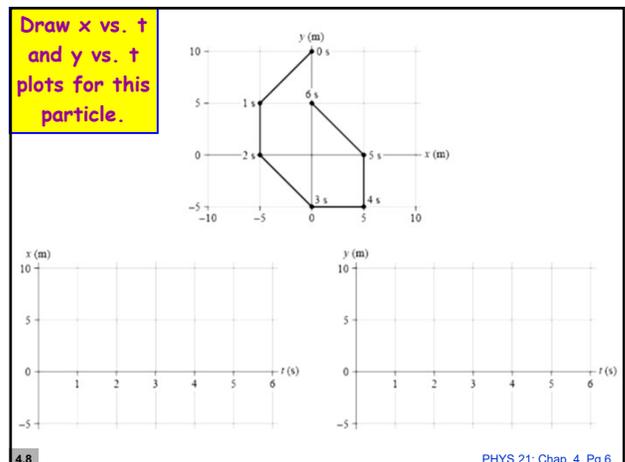
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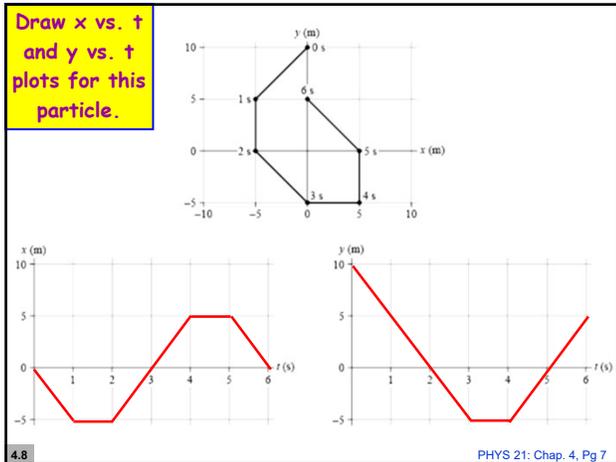
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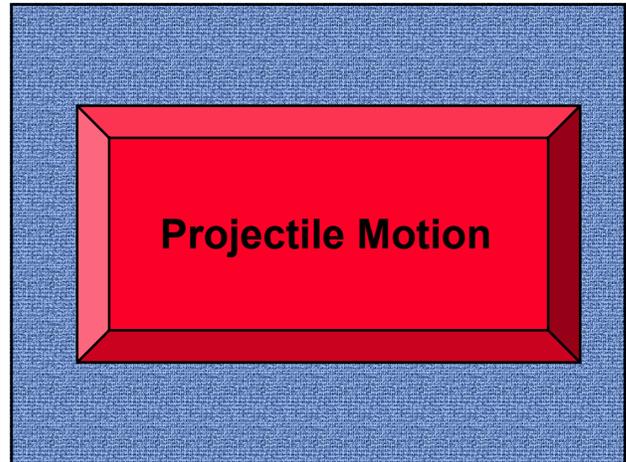
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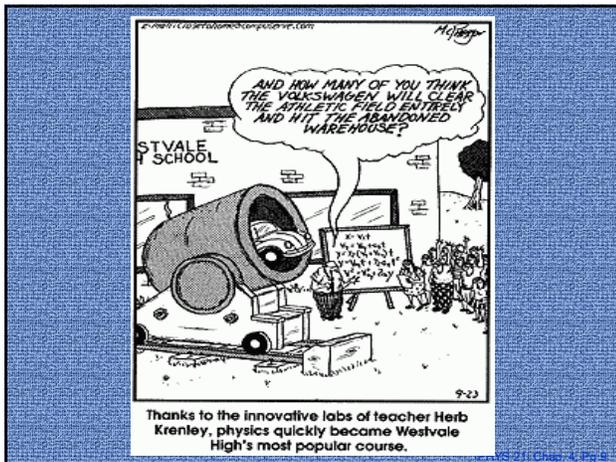
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A projectile is launched over level ground and lands some distance away.

a) Is there any point on the trajectory where the velocity v and acceleration a are **parallel** to each other? If so, where? no

b) Is there any point on the trajectory where the velocity v and acceleration a are **perpendicular** to each other? If so, where? yes

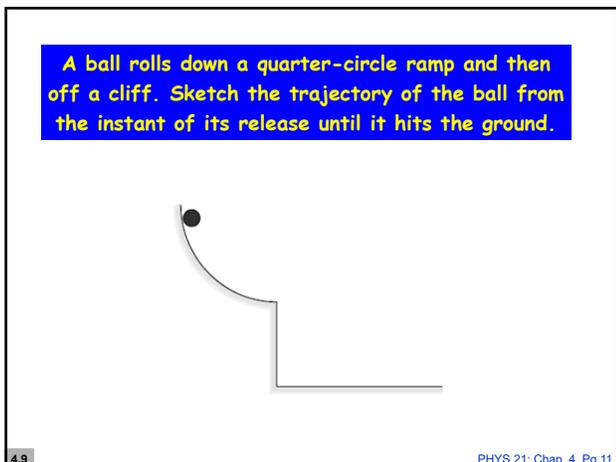
c) Which of the following quantities are **constant** throughout the flight?

x y v v_x v_y a_x a_y

3.30

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ConceptTest 1 **Projectile Motion**

A ball rolls horizontally off a table, and at the same instant, a second ball is simply dropped from the table? Which ball hits the ground first?

1. the rolling ball hits first
2. the dropped ball hits first
3. both hit at the same time
4. I really have no idea

Vertical Motion Only

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A ball rolls down a quarter-circle ramp and then off a cliff. Sketch the trajectory of the ball from the instant of its release until it hits the ground.

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Let's say that the ball takes **5 s** to fall to the ground. At each interval, draw the vectors for v_x and v_y . Label them with the numerical value of the components at that point.

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Let's say that the ball takes **2 s** to fall to the ground. At each interval, draw the vectors for v_x and v_y . Label them with the numerical value of the components at that point.

$g = 9.8 \text{ m/s}^2$

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SYNTHESIS 3.1 Projectile motion

The horizontal and vertical components of projectile motion are independent, but must be analyzed together.

An object is launched into the air at an angle θ to the horizontal with initial speed v_i .

After launch, the horizontal motion is uniform motion. The horizontal component of the initial velocity is the initial velocity for the horizontal motion. The acceleration is zero.

After launch, the vertical motion is free fall. The vertical component of the initial velocity is the initial velocity for the vertical motion. Rising or falling, the acceleration is the same, $a_y = -g$.

The kinematic equations for projectile motion are those for constant-acceleration motion vertically and constant-velocity motion horizontally:

The vertical motion is free fall. The free-fall acceleration, $g = 9.8 \text{ m/s}^2$.

The horizontal motion is uniform motion.

$(v_x)_t = (v_x)_i = \text{constant}$

$x_t = x_i + (v_x)_i \Delta t$

The components of the initial velocity are found from trigonometry. $(v_x)_i = v_i \cos \theta$ $(v_y)_i = v_i \sin \theta$

The two equations are linked by the time interval Δt , which is the same for the horizontal and vertical motions.

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ConceptTest 2 Rolling Cart

A cart rolling along at constant speed fires a ball straight up. When the ball comes back down, where will it land?

- in front of the tube
- behind the tube
- directly in the tube
- it will not come down

Will the answer change if the cart is accelerating in the forward direction?

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x and y motions are independent

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ConcepTest 2a Rolling Cart on Incline

The same cart is now rolling down a ramp and shoots the ball out of the tube. Now where does the ball land when it comes back down?

1. in front of the tube
2. behind the tube
3. directly in the tube
4. depends on the speed of the cart

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x and y motions are independent

Kinematic Equations for Constant Acceleration in 2 Dimensions

x Component (horizontal)	y Component (vertical)
$v_x = v_{x0} + a_x t$	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

Projectile Motion: $a_x = 0$ and $v_x = \text{const.}$
 (y positive upward; $a_x = 0$, $a_y = -g = -9.80 \text{ m/s}^2$)

Horizontal Motion ($a_x = 0$, $v_x = \text{constant}$)	Vertical Motion [†] ($a_y = -g = \text{constant}$)
$v_x = v_{x0}$	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0} t$	$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$
	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

[†] If y is taken positive downward, the minus (-) signs in front of g become plus (+) signs.

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PROBLEM-SOLVING APPROACH 3.1 Projectile motion problems

- 1 STRATEGIZE** We will solve projectile motion problems by considering the horizontal and vertical motions as separate but related problems.
- 2 PREPARE** There are a number of steps that you should go through in setting up the solution to a projectile motion problem:
 - Make simplifying assumptions. Whether the projectile is a car or a basketball, the motion will be the same.
 - Draw a visual overview including a pictorial representation showing the beginning and ending points of the motion.
 - Establish a coordinate system with the x-axis horizontal and the y-axis vertical. In this case, you know that the horizontal acceleration will be zero and the vertical acceleration will be free fall: $a_x = 0$ and $a_y = -g$.
 - Draw a vector representing the initial velocity, and find its x- and y-components in terms of the initial speed and the launch angle.
 - Define symbols and write down a list of known values. Identify what the problem is trying to find.
- 3 SOLVE** There are two sets of kinematic equations for projectile motion, one for the horizontal component and one for the vertical:

Horizontal	Vertical
$x_f = x_i + (v_x)_i \Delta t$	$y_f = y_i + (v_y)_i \Delta t - \frac{1}{2} g (\Delta t)^2$
$(v_x)_f = (v_x)_i = \text{constant}$	$(v_y)_f = (v_y)_i - g \Delta t$
- 4 ASSESS** Check that your result has the correct units, is reasonable, and answers the question.

Δt is the same for the horizontal and vertical components of the motion. Find Δt by solving for the vertical or the horizontal component of the motion; then use that value to complete the solution for the other component.

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Projectile Motion

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Four balls are launched at the same speed from height h. Simultaneously, ball 5 is dropped from rest at the same height.

Rank in order, from shortest to longest, the time it takes each of the balls to hit the ground.

4 < 3 < (2=5) < 1

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Rank in order, from shortest to longest, the time it takes each of these balls to hit the ground.

$(1=2=3=4) < 5$

$1 < (2=4) < 5 < 3$

Rank in order, from shortest to longest, the horizontal distance that each of these balls travels before hitting the ground.

3.34

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ConceptTest 3 Projectile Motion

Rank in order, from longest to shortest, the time in the air for each of these kicks.

1. $1 > 2 > 3$
2. $1 = 2 = 3$
3. $3 > 2 > 1$

Follow-up: how would you rank the initial launch speeds of the kicks?

$3 > 2 > 1$

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ConceptTest 4 Projectile Motion

A battleship simultaneously fires two shells at enemy ships, and the trajectories are shown. Which ship gets hit first?

1. ship A
2. ship B
3. both at the same time

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complementary angles give the same range

$v_i = 50 \text{ m/s}$

maximum range occurs at 45°

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The longest recorded pass in an NFL game traveled **83 yds** in the air from the quarterback to the receiver. If the pass was thrown at a launch angle of 45° , what was the speed at which the ball left the quarterback's hand?

$x = 27.3 \text{ m/s}$

3+2=5

3.64

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29

The longest recorded pass in an NFL game traveled **83 yds** in the air from the quarterback to the receiver. If the pass was thrown at a launch angle of 45° , what was the speed at which the ball left the quarterback's hand?

$x = 27.3 \text{ m/s}$

$$v_{ix} = v(\cos\theta) \quad v_{iy} = v(\sin\theta)$$

$$v_{fy} = -v_{iy} = v_{iy} - gt \quad \Rightarrow \quad t = \frac{v_{fy} - v_{iy}}{-g} = \frac{-2v(\sin\theta)}{-g}$$

$$\Delta x = v_{ix}t = v(\cos\theta)t = \frac{2v^2(\cos\theta)(\sin\theta)}{g} = 83 \text{ yds} = 76 \text{ m}$$

$$v^2 = \frac{g(76 \text{ m})}{2(\cos 45^\circ)(\sin 45^\circ)} \quad \Rightarrow \quad v = \sqrt{\frac{g(76 \text{ m})}{2(\cos 45^\circ)(\sin 45^\circ)}} = 27.3 \text{ m}$$

3+2=5

3.64

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